Stock-Outs and Customer Purchasing Behavior when Product Quality is Uncertain

Laurens G. Debo
Garrett J. van Ryzin

University of Chicago, Columbia University

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Where's Elmo? Toy remains hard to find

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By Nicole Maestri

NEW YORK (Reuters) - One month since T.M.X. Elmo first laughed, slapped his knee and keeled over onto U.S. shoppers' stage, consumers' efforts to secure one of the now-scarce furry red dolls has become increasingly difficult.
Long Queues

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More Stories of Shortages

WSJ, Dec 2, 2005, *Why shortages of Hot gifts endure as a Christmas ritual*

<table>
<thead>
<tr>
<th>Year</th>
<th>Product</th>
<th>Year</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983</td>
<td>Cabbage Patch Kids</td>
<td>2000</td>
<td>Play station 2</td>
</tr>
<tr>
<td>1990s</td>
<td>Beanie Babies</td>
<td>2002</td>
<td>Nike Airforce1</td>
</tr>
<tr>
<td>1998</td>
<td>Tickle me Elmo</td>
<td>2004</td>
<td>iPod mini, Nitendo DS</td>
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<tr>
<td>1999</td>
<td>Pokeman</td>
<td>2005</td>
<td>iPod nano</td>
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Mighty Morphin Power Rangers, shortage in 1994
What explains these shortages?

- “Newsvendor logic” – High demand uncertainty combined with risks of overage and underage; firms simply can’t meet all demand under all conditions. These are just examples of “losing the bet” on the upside.

- They’re deliberately created – Shortages create buzz, hype, a sense of urgency. . . . shortages boost demand, so firms have an incentive to deliberately limit supply.
The basic mechanism

- Inventories convey information about other consumer’s decisions, which influence their buying decision.
- Consumers observe stock-outs.

SO

- Inventories impact customers’ purchasing decisions.
- Customers’ purchasing decisions impact inventories.
- Equilibrium outcome determines realized sales.
Key Questions

- How do customers learn from stock-outs, and how does that learning influence total sales?
- How does the inventory investment and allocation to retailers impact their purchasing behavior?
- How should a firm take this consumer behavior into account when investing in inventory and allocating it to retailers?
Outline

1. Introduction and Motivation
2. The Model
3. Analysis of a Single Retailer
   - Consumer purchasing behavior
   - Inventory investment
4. Analysis of a Two retailers Retailer
   - Consumer purchasing behavior
   - Inventory Investment
5. Conclusion
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The Model: Key ingredients

- The product is short-lived, and its true (common) quality is not revealed during the season.
- Two retailers; consumers arrive randomly at these two retailers; sales are lost if there is no inventory.
- Consumer quality-related information:
  - Common prior on product quality
  - Private: private signal about realized quality
  - Some consumers observe how many retailers are out of stock (informed)
  - Some consumers do not observe how many retailers are out of stock (uninformed)
The Model: Key ingredients

- Quality $v_\omega$, where $\omega \in \{\ell, h\}$ is unknown and $v_\ell < 0 < v_h$, where $v_h = -v_\ell = v$ with prior $\Pr(\omega = h) = p_0$.

- Private a signal $s \sim G_\omega(s)$. $s \in [s, \bar{s}]$, $\frac{g_h(s)}{g_\ell(s)}$ is increasing, $g_h(s) = g_\ell(\bar{s}) = 0$.

- Retailer inventory $Q_1 = Q$ and $Q_2 = Q + \Delta$. $I = (Q, \Delta)$.

- Mass $\lambda$ of potential consumers.

- A fraction $\alpha \in (0, 1)$ observes only a private signal, a fraction $1 - \alpha$ also observe the number of stocked out retailers, $m \in \{0, 1, 2\}$.

- All consumers can switch at no cost from one retailer to the other retailer.
Overview of the model

Potential Market
- \( u = E[v_\omega] = \text{utility} \)
- \( \omega = l \text{ or } h \)
- \( v_l > 0 > v_r \)

\( \lambda \)

\( \alpha: a^{il} = 1 \text{ iff } u^{il}(s, I) > 0 \text{ & } m < 2 \)

\( 1 - \alpha: a^{i} = 1 \text{ iff } u^{i}(s, m, a, I) > 0 \text{ & } m < 2 \)

- \( m = \text{ number of stock-outs} \)
- \( s = \text{ private signal} \sim \omega \)

Retailer

Firm

\( Q_1 \)

\( Q_2 \)

\( r = \text{ revenue} \)

\( c = \text{ cost} \)

\( I = \text{Inventory} \)
The Model: Key ingredients

Equilibrium Conditions:

1. $u^u(s)$ is uninformed consumer utility

$$u^u(s, I) > 0 \Rightarrow \text{buy}$$

2. $u^i(m, s, s^*, I)$ is informed consumer utility

$$u^i(m, s, s^*, I) > 0 \Rightarrow \text{buy}$$

3. $\Pi(s, I)$ is the firm’s expected profit

$$\Pi^* = \max_{I \in \mathbb{R}_+^2} \Pi(s^*(I), I)$$
Analysis of a Single Retailer

- $Q = 0$ (no small retailer)
- $\Delta$ is total inventory at large retailer
Uninformed Consumers

- Updated utility:

\[ u^U(s) = p'(s)v + (1 - p'(s))(-v) \]

- Updated prior on product quality:

\[ p'(s) = \frac{g_h(s)p_0}{g_h(s)p_0 + (1 - p_0)g_\ell(s)} \]

- Purchasing threshold: purchase if private signal is larger than \( \hat{s} \):

\[ l(\hat{s})\theta = 1, \]

where \( l(s) = \frac{g_h(s)}{g_\ell(s)} \) and \( \theta = \frac{p_0}{1-p_0} \).
Updated utility:

\[ u^i (m, s, s_0, \Delta) = p'' (m, s, s_0, \Delta) v + (1 - p'' (m, s, s_0, \Delta)) (-v) \]

Updated prior on product quality:

\[ p'' (m, s, s_0, \Delta) = \frac{g_h (s) p_0 p^h (m, s_0, \Delta)}{g_h (s) p_0 p^h (m, s_0, \Delta) + (1 - p_0) g_\ell (s) p^\ell (m, s_0, \Delta)} \]

where \( p^{\omega} (m, s_0, \Delta) \) is the probability of observing \( m \) stock-outs
Informed Consumers

- Probability of observing \( m \) stock-outs: \( p^\omega (m, s_0, \Delta) \).
  - Quality-dependent purchasing probability:
    \[
    P^\omega (s) = \alpha \overline{G}_\omega (\hat{s}) + (1 - \alpha) \overline{G}_\omega (s).
    \]
  - Volume of consumers observing no retailer out of stock:
    \[
    \overline{\lambda}^\omega (s_0, \Delta) = \frac{\Delta}{P^\omega (s_0)}.\]
  - Probability of observing \( m \) stock-outs:
    \[
    p^\omega (0, s_0, \Delta) = \min \left( 1, \frac{\overline{\lambda}^\omega (s_0, \Delta)}{\lambda} \right), \quad p^\omega (1, s_0, \Delta) = 1 - p^\omega (0, s_0, \Delta)
    \]
Purchasing threshold: purchase if private signal is larger than $s_0^*(\Delta)$, which solves:

$$\theta I(s_0) = \frac{p^e(0, s_0, \Delta)}{p^h(0, s_0, \Delta)}.$$  (1)
Equilibrium Purchasing Threshold $s_0^*$

$p_0 = 1/2 \rightarrow \theta = 1$
Inventory Investment Problem

- Expected satisfied sales:

\[ S(\Delta, s_0) = \mathbb{E}_\omega [\min (\Delta, P^\omega (s_0) \lambda)] \]

- Expected profits:

\[ \Pi(\Delta, s_0) = rS(\Delta, s_0) - c\Delta \]

- Optimal Inventory:

\[ \max_{\Delta \in [0, \lambda]} \Pi(\Delta, s^*_0(\Delta)) \]
Baseline: Inventory Investment for All Uniformed Consumers

\[ \max_{\Delta \in [0, \lambda]} rS(\Delta, \hat{s}) - c\Delta \]

- \( P^h (\hat{s}) \lambda \) is optimal when \( p_0 > \frac{c}{r} \). High margin: stock out if high quality only
- \( P^\ell (\hat{s}) \lambda \) is optimal when \( p_0 < \frac{c}{r} \). Low margin: stock out if both low and high quality
Inventory Investment Problem: Informed Consumers

There exist $\hat{c} > c$ and $\tilde{s} > \hat{s}$:

- $P^h(\hat{s}) \lambda$ is optimal when $p_0 > \frac{\hat{c}}{r}$.
- $P^l(\tilde{s}) \lambda < P^l(\hat{s})$ is optimal when $p_0 < \frac{\hat{c}}{r}$.

Conclusion:

- Expanded range of ‘high margin’ products
- Low-quality realization has less demand
- Low-margin inventory investment is lower
Two Retailers

- Two retailers: small and large
- \( Q_1 = Q \) and \( Q_2 = Q + \Delta \)
- Stock-out signal \( m \in \{0, 1, 2\} \)
Uninformed and Informed Consumers

- Uninformed Consumers: same as for the Single Retailer Case.

- Informed Consumers: Key is \( p^\omega (m, s, I) \) is the probability of observing \( m \) stock-outs.

- Volume of consumers observing \( m \) retailers out of stock:

\[
\lambda^\omega (s, I) = \frac{Q}{\frac{1}{2} P^\omega (s_0)} \quad \text{and} \quad \bar{\lambda}^\omega (s, I) = \frac{Q}{\frac{1}{2} P^\omega (s_0)} + \frac{\Delta}{P^\omega (s_1)}.
\]
Inventory Depletion

\[ Q + \Delta \]

\[ \lambda \omega \]

\[ P_\omega(s_0)/2 \]

\[ P_\omega(s_1) \]

\[ P_\omega(s_0)/2 \]

\[ \lambda_\omega \]

\[ \overline{\lambda}_\omega \]

\[ \lambda \]

\[ \text{Market potential} \]
Uninformed and Informed Consumers

- **Probability of observing** $m$ **stock-outs:**

  \[
  p^\omega(0, s_0, I) = \min \left(1, \frac{\lambda^\omega(s_0, I)}{\lambda}\right) \quad \text{and} \\
  p^\omega(1, s, I) = \min \left(1, \frac{\lambda^\omega(s, I)}{\lambda}\right) - \frac{\lambda^\omega(s_0, I)}{\lambda}
  \]

  for $1 \geq \frac{\lambda^\omega(s_0, I)}{\lambda}$ (and 0 otherwise).

- **Purchasing threshold:** purchase if private signal is larger than $s^*(I)$:

  \[
  \theta_l(s^*_m) = \frac{p^\ell(m, s^*, I)}{p^h(m, s^*, I)}, \ m \in \{0, 1\}. \tag{2}
  \]
Small Retailer: End-of-Season-Inventory

\((\kappa = 0.125, p_0 = 0.45, \alpha = 0.25)\)
Large Retailer: End-of-Season-Inventory

\( \kappa = 0.125, \rho_0 = 0.45, \alpha = 0.25 \)
Inventory Investment Problem

- Expected satisfied sales:

\[
S(I, s) = \mathbb{E}_\omega [2 \min(Q, \frac{1}{2} P^\omega (s_0) \lambda) \\
+ \min(\Delta, P^\omega (s_1) (\lambda - Q/(\frac{1}{2} P^\omega s_0))^+) ]
\]

- The impact of Inventory on satisfied sales:

\[
\frac{\partial}{\partial I} S(I, s^*(I)) + \frac{\partial}{\partial s_0} S(I, s^*(I)) \frac{\partial s^*_0}{\partial I} + \frac{\partial}{\partial s_1} S(I, s^*(I)) \frac{\partial s^*_1}{\partial I}.
\]

for \( I = Q \) or \( \Delta \)
Inventory Investment Problem

- Expected profits:

$$\Pi(l, s) = rS(l, s) - c(2Q + \Delta)$$

- Optimal Inventory:

$$\max_{l \in \mathbb{R}^2_+} \Pi(l, s^*(l))$$
Optimal Inventory: Total Inventory Investment

\( \kappa = 0.125, \ p = 0.45, \ \alpha = 0.25 \)
Optimal Inventory: Inventory Allocation

\((\kappa = 0.125, p = 0.45, \alpha = 0.25)\)
Optimal Inventory: Profits ($\kappa = 0.125, \rho = 0.45, \alpha = 0.25$)
For high margin products with a low prior, the optimal inventory investment is higher than the uninformed inventory (capture the herd!)

Mostly, small small retailers are optimal (no perfect high quality communication through stock-outs).
The profit increase with respect to the uninformed profits can be substantial (30%), especially for high margin products with a low prior and noisy private signals.

For very low margin products it is optimal to reduce the inventory investment below the uninformed inventory (forgo high demand option, waste less informed consumers).
Conclusion

- Increase sales with stock-outs?
- Requires asymmetric inventory allocation.
- Example: Apple introduces iPod at BestBuy and Apple Stores

Extensions

- Stochastic demand
- Search costs
- Different retailer margins
- Laboratory or empirical research