Pricing with Markups under Horizontal and Vertical Competition

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Nicolás Stier-Moses, Columbia Business School
stier@gsb.columbia.edu

Joint work with
José Correa, Universidad de Chile
Roger Lederman, Columbia Business School
Motivation for supply function equilibria (SFE)

- Early commitment to prices or quantities limit the possibilities to adjust

  - **Price functions** model that producers postpone decisions until demand is realized

- More robust to uncertainty in demand (arising from market variations or from impossibility to predict perfectly what competitors will do)

- SFE has been applied to markets of substitutes

- Typical examples: electricity, airlines, consulting, …
This talk: SFE allowing complements

- Producers compete for consumers that are looking to purchase a bundle of products

- How does a producer price depending on the number and efficiency of competitors offering
  - substitutes (alternatives to its offering)?
  - complements (other items in the bundle)?
Each path from origin \( s \) to destination \( t \) represents a feasible bundle, where each link is a producer.

We consider a series-parallel structure, defined recursively by adding links in series or in parallel.

Significantly generalizes existing literature on supply function equilibria [Klemperer & Meyer ‘89, Akgun ’04]
Overview of Game

We consider a pricing game with two stages:

1. Producers learn their cost functions
2. Producers decide their price functions playing a non-cooperative perfect information game
3. Consumer(s) learn price functions of producers
4. Consumer(s) decide what bundle to buy playing another perfect information game
Questions

- What are good strategies for producers? Are resulting prices similar to costs?
- Does there exist a price function equilibrium?
- Is it unique?
- How can we compute it?
- How efficient is the resulting assignment compared to one in which consumers pay production costs?
Main Contributions

- Compute price function equilibrium for series-parallel markets
- There is equilibrium s.t. \( \text{price} = \text{cost} + \text{markup} \)
- Characterize the degree of competition as a property of the network structure
- As long as there is enough competition, there is a limit to the inefficiency introduced by price distortions
Is it natural to use markups?

- It is common in practice, but also:
  - Any equilibrium in markups is also an equilibrium in the space of non-decreasing price functions.

- In the case of stochastic demand and substitutes, linear costs imply a unique equilibrium with linear price functions.

  This is a markup equilibrium  

  [Klemperer & Meyer’89]
The Model: Producers

- Producers’ *unit cost functions are linear but heterogeneous*: slope = efficiency parameter $c_a$

- Hence, for a demand $x_a$ for producer $a$, per-unit cost is $c_a x_a$ and total cost is $c_a x_a^2$

- **Cost-plus**: producer $a$ chooses a markup $\alpha_a$ so price function = cost function $\times$ markup

- Total revenue for producer $a$ is $\alpha_a c_a x_a^2$
The Model: Consumers

- Consumers are price takers. Deterministic and inelastic demand is normalized to $D = 1$

- Once price functions are set, consumers clear the market buying the bundle at the lowest price

- The 2\textsuperscript{nd} stage game amounts to consumers choosing the cheapest path through a network
Simple Example with Substitutes

- Three producers with unit production costs equal to $x$, $2x$ and $2x$
- Each producer solves

$$\max(\alpha_a - 1)(x_a^{\text{NE}})^2$$

where

$$x_a^{\text{NE}} := \frac{1/(\alpha_a c_a)}{\sum_i 1/(\alpha_i c_i)}$$
The Equilibrium in the Example

An equilibrium simultaneously solves:

\[
\begin{align*}
\max(\alpha_1 - 1)\alpha_1 & \left(\frac{1}{\alpha_1 + 1/2\alpha_2 + 1/2\alpha_3}\right)^2 \\
\max(\alpha_2 - 1)2\alpha_2 & \left(\frac{1/2\alpha_2}{\alpha_1 + 1/2\alpha_2 + 1/2\alpha_3}\right)^2 \\
\max(\alpha_3 - 1)2\alpha_3 & \left(\frac{1/2\alpha_3}{\alpha_1 + 1/2\alpha_2 + 1/2\alpha_3}\right)^2
\end{align*}
\]

Leading to:

\[\alpha_1 = 5.56; \quad \alpha_2 = \alpha_3 = 3.56\]

\[x_1 = 0.39, \quad x_2 = x_3 = 0.305\]

Total production cost = 0.525
An optimal allocation is one minimizing total production cost

\[
\min \left( x_1^2 + 2(x_2)^2 + 2(x_3)^2 \right)
\]
such that \( \sum x_a = 1 \)

Solution: \( x_1 = 1/2; x_2 = x_3 = 1/4 \)

Total production cost = 0.5

Hence, price distortions generate an efficiency-loss of 5%
\( (= 0.525/0.5) \)
Consumers allocation to bundles minimizes the convex objective function:

\[ \sum_{a \in A} \left[ \int_{0}^{x_{a}^{NE}} \alpha_{a} c_{a} u \, du \right] \]

For any two bundles \( B_1 \) and \( B_2 \), at equilibrium:

\[ \sum_{i \in B_1} \alpha_{i} c_{i} x_{i}^{NE} = \sum_{j \in B_2} \alpha_{j} c_{j} x_{j}^{NE} \]

Works with single consumer, who minimizes

\[ \sum_{a \in A} \alpha_{a} c_{a} x_{a}^{2} \]

[Beckmann et al. '56]
For a given vector $\alpha$, the price paid to producer $a$ is $x_a$ times a price multiplier of $\alpha_a c_a$.

Prices of substitutes are equal at equilibrium.

The price paid for a bundle is $(R \times \text{demand})$, where $R$ is defined recursively from the price multipliers of individual producers:

$$R_{\text{series}} = \alpha_1 c_1 + \alpha_2 c_2$$

$$R_{\text{parallel}} = \frac{1}{\alpha_1 c_1} + \frac{1}{\alpha_2 c_2}$$
For each producer $a$, we define a term $R_a$ to describe the price function for all market minus it.

Solving for the equilibrium allocation, $R_a$ determines the **elasticity** of residual demand for producer $a$:

$$x_{a}^{NE} = \mu_a \left( \frac{1}{1 + \frac{\alpha_a c_a}{R_a}} \right)$$
Maximizing the profit function leads to an optimal markup $\alpha_a$ as a best response to others’ markups:

$$\alpha_a c_a = 2c_a + R_a$$

An equilibrium is a vector $\alpha$, where this relation is satisfied simultaneously for all producers.
What is driving producer markups?

- Markups **decrease** as competitors are added in parallel.

- Markups **increase** in the number of producers placed in series.

- Decrease again as competitors are added downstream.

*Increasing function of all other producers’ markups*
Equilibria of the Game

- Previous system of equations cannot be solved in closed form
- Have complete characterization and algorithm for case of substitutes
  [Correa, Figueroa, S.-M.’09]
- In general, doing **iterated best responses** produces **increasing markups** at successive rounds
- If markups are bounded, sequence converges and limit has to be an equilibrium
• Hence, can prove existence by proving that markups are bounded

• An equilibrium **exists** iff the undirected graph modeling the network is **3-edge-connected**

• Equilibrium is **unique** when it exists

*Intuition*: not enough competition for two critical producers so iterated best responses produces unbounded markups
Observations: causes of instability

- Duopoly competitors will continually increase their markups above competitor

- This behavior is also observed by two producers with no competition along a particular vertical
Producers *merge* to form a single producer with an equivalent aggregate cost structure

- Horizontal mergers *increase* the price of a bundle

\[
\frac{c_1 c_2}{c_1 + c_2}
\]

- Vertical integration *decreases* the price of a bundle

\[
\frac{c_1 c_2}{c_1 + c_2} + c_3
\]

*Prices for individual products may increase or decrease in either case, depending on their relative position in the market.*
Define the total social cost of an allocation $x$ by:

$$C(x) = \sum_{a \in A} c_a x_a^2$$

When an upper bound $\bar{\alpha}$ exists for producer markups, the inefficiency of an equilibrium is bounded:

$$\frac{C(\text{equilibrium})}{C(\text{social optimum})} \leq \frac{\bar{\alpha}}{2} \leq \frac{1}{1 - \max_a \{\sigma_a\}}$$

Where $\sigma_a = \frac{C_a}{c_a}$, defining $C_a$ as for $R_a$ with cost not marked up, is a measure of producer $a$’s market power.
Inefficiency in the case of substitutes

![Graph showing price of anarchy vs. sigma with lines for lb poa, ub poa, and gap]

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Conclusions

● We define market power in terms of producer’s position in the network representation of the market.

● Competition limits markups applied by producers. Price distortions are capped.

● With sufficient competition, unique equilibrium markups exist and can be computed.

● We can study the dynamics of vertical competition.
Extensions

We are currently looking at:

- More general network structures
- Producers controlling multiple links
- Cost functions of other shapes (arbitrary-degree polynomials)
- Elastic demand
Thank You
Observations: other network structures

- When Series-Parallel restriction is violated, consumers may not buy from some producers.

- As a result, 3-edge-connectedness is not sufficient for existence of equilibrium.

There is no equilibrium for which the middle link is used. Removing the link reduces connectedness.
Marginally increasing per-unit production cost $c_\alpha u(x_\alpha)$, that depends on quantity produced.

Standard assumption in economics. E.g.:

- More demand implies more willingness to pay
- Capacities
- Labor: additional shifts cost more (regular hours $\rightarrow$ overtime $\rightarrow$ temps)
3-edge-connected graph

- Definition: “There is no pair of edges whose removal disconnects the graph”

*Intuition*: a producer’s market power is tied to its importance in maintaining connectivity of the underlying graph

- Can efficiently detect 3-edge-connectedness using max flow algorithm